Appendix F

Theoretical Rate Calculation for Scenario 2

Scenario 2 has a well-defined set of traffic flows in a network whose links have fixed bandwidths. It should be possible to determine a set of rates for each flow which ensures the highest link utilisation, where possible, and also divides the link capacity fairly amongst the flows.

In Scenario 2, the bottleneck in the system is the 64kbps full-duplex link 2 $\leftrightarrow$ 3. We will assume that all acknowledgment rates are limited by their respective data rates by

$$A_x = R * D_x, \ where \ R = \frac{ack \ pkt \ size}{data \ pkt \ size} = \frac{40}{1,500} \tag{F.1}$$

where D_x denotes a data flow from source x to its destination and A_x denotes the corresponding acknowledgment flow in the reverse direction.

At time $t = 0$, the flows D_5 and A_5 commence. The optimum rates for both are:

Time	Optimum Rate (bps)	
	D_5	A_5
$t \geq 0$	64,000	1706

At time $t = 12$, the flows D_7 and A_7 commence. As the link 2 $\leftrightarrow$ 3 is the bottleneck, we can write limiting equations for flows across the link 2 $\rightarrow$ 3 and across the link 3 $\rightarrow$ 2:

$$D_5 + D_7 \leq 64,000 \tag{F.2}$$

$$A_5 + A_7 \leq 64,000 \tag{F.3}$$

(2) is the limiting equation, as the data flows are substantially larger than the acknowledgment flows. By sharing bandwidth equally to the flows, we obtain the rates:

Time	Optimum Rate (bps)			
	D_5	A_5	D_7	A_7
$t \geq 12$	32,000	853	32,000	853

At time $t = 18$, the flows D_8 and A_8 commence. Here, the flows are in the opposite directions to the previous four flows. Again, we can write limiting equations for the flows:

$$D_5 + D_7 + A_8 \leq 64,000 \tag{F.4}$$
$$A_5 + A_7 + D_8 \leq 64,000 \tag{F.5}$$

Distributing bandwidth fairly, we have $D_5 = D_7$ and $A_5 = A_7$. Assume both links are fully utilised and substituting for the acknowledgment rates using (1), we have:

$$2D_5 + R * D_8 = 64,000 = 2R * D_5 + D_8 \tag{F.6}$$
$$2D_5(1 - R) = D_8(1 - R) \tag{F.7}$$
$$2D_5 = D_8 \tag{F.8}$$

Substituting (8) into (6) gives $D_5 = 31,168$, $D_8 = 62,337$ and the table:

Time	Optimum Rate (bps)					
	D_5	A_5	D_7	A_7	D_8	A_8
$t \geq 18$	31,168	831	31,168	831	62,337	1,662

The six flows continue to transmit until time $t = 140$ when the flows D_6 and A_6 commence. Again, distributing bandwidth fairly, we have $D_5 = D_7 = D_6$ and $A_5 = A_7 = A_6$. Assume both links are fully utilised and substituting for the acknowledgment rates using (1), we have:

$$3D_5 + R * D_8 = 64,000 = 3R * D_5 + D_8 \tag{F.9}$$
$$3D_5(1 - R) = D_8(1 - R) \tag{F.10}$$
$$3D_5 = D_8 \tag{F.11}$$

Substituting (11) into (9) gives $D_5 = 20,779$, $D_8 = 62,337$ and the table:

Time	Optimum Rate (bps)							
	D_5	A_5	D_7	A_7	D_8	A_8	D_6	A_6
$t \geq 140$	20,779	554	20,779	554	62,337	1,662	20,779	554

As flow D_6 only has 100 packets to send, it finishes quickly, and the rates return to the values before flow D_6 started:

Time	Optimum Rate (bps)					
	D_5	A_5	D_7	A_7	D_8	A_8
$t \geq Off_6$	31,168	831	31,168	831	62,337	1,662

Similarly, flow D_8 has most of the bandwidth across the link 3 $\rightarrow$ 2, so it is the next to finish, and the remaining flows return to the rate values before flow D_8 started:

Time	Optimum Rate (bps)			
	D_5	A_5	D_7	A_7
$t \geq Off_8$	32,000	853	32,000	853

The flows D_5, D_7, A_5 and A_7 continue at these rates until one or both finish.